CATEGORIES IN ABSTRACT MODEL THEORY

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We highlight the category-theoretic bent of recent work in abstract model theory, particularly that involving abstract elementary classes (AECs), and make a connection with a subject of current research within category theory—accessible categories.

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We also address the following questions:

- Can we find meaningful analogues/translations of AEC notions in the category-theoretic framework? Categoricity? Stability?
- Does the shift in perspective yield model-theoretic dividends?

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Problem: elementary classes—classes of models of such theories—do not exhaust the interesting classes of mathematical objects.

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For the purposes of this talk, abstract model theory is the research program focused on sniffing out the fragment of classification theory that is common to all such classes.

To achieve that end, we cannot work piecemeal—logic by logic—as results do not generalize well.

Strategy: abandon syntax and logic-dependent structure entirely, and simply work with abstract classes of structures equipped with a strong substructure relation that retains certain essential properties of elementary embedding.

Hence abstract elementary classes—which can (and perhaps should) be regarded as the category-theoretic hulls of elementary classes.

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A strong embedding $f : M \hookrightarrow_{\mathcal{K}} N$ is an isomorphism from M to a strong submodel of N, $f : M \cong M' \prec_{\mathcal{K}} N$.

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Having discarded syntax, we consider a new notion of type: Galois types. In AECs with sufficient amalgamation, there is a monster model $\mathfrak{C}$ in $\mathcal{K}$, and the Galois types have a simple description:

Definition

For $a \in \mathfrak{C}$, $M \in \mathcal{K}$, the *Galois type of a over M* is defined to be the orbit of a under automorphisms of $\mathfrak{C}$ that fix M. The set of all types over M is denoted by ga-S(M).

In general, they will not have a nice syntactic description.

Definition

We say $\mathcal{K}$ is λ-*Galois-stable* if $|\text{ga-S}(M)| = \lambda$ for all $M \in \mathcal{K}_\lambda$.

Some familiar notions translate:

Definition

A model $M \in \mathcal{K}$ is λ-Galois-saturated if it realizes all types over submodels of size less than λ, i.e. the orbit in $\mathfrak{C}$ corresponding to any such type meets M.

Some notions are less familiar:

Definition

An AEC $\mathcal{K}$ is χ-tame if, for any distinct Galois types p and q over a model $M \in \mathcal{K}$, there is $N \prec_{\mathcal{K}} M$ with $|N| \leq \chi$ such that $p \upharpoonright N \neq q \upharpoonright N$.

Categoricity: Story fragmentary, results only for tame classes.

Theorem (Grossberg-VanDieren)

If $\mathcal{K}$ is categorical in λ and λ^+, it is categorical in λ^{++}.

Theorem (G-V)

If $\mathcal{K}$ is categorical in $\lambda^+ > H(\mathcal{K})$, it is categorical in all $\mu > H(\mathcal{K})$.

These results involve resorting, however briefly, to syntactic considerations.

Question: Can a purely category-theoretic perspective reveal anything new about the structure of categorical AECs?

Stability: For tame AECs, some progress (G-V, Baldwin-Kueker-V, L). If $\mathcal{K}$ is only weakly tame, very little:

Theorem (B-K-V)

If $\mathcal{K}$ is λ-stable, it is stable in λ^{+n} for all $n < \omega$.

Theorem (L)

If $\mathcal{K}$ is λ-t.t., and κ is such that cf(κ) $> \lambda$ and each M of size κ has a saturated extension also of size κ, then $\mathcal{K}$ is κ-stable.

Can we guarantee the existence of saturated extensions without making the standard model-theoretic assumption: $|\text{ga-S}(M)| < \kappa$ for all $M \in \mathcal{K}_{<\kappa}$?

Yes: weak κ-stability, a purely category-theoretic (and weaker) notion, will suffice.

Roughly speaking, an accessible category is one that is generated by colimits of a set of small objects. To be precise:

Definition

An object N in a category $\mathbf{C}$ is λ-presentable if the functor $\mathrm{Hom}_{\mathbf{C}}(N, -)$ preserves λ-directed colimits.

Definition

A category $\mathbf{C}$ is λ-accessible if

- it has at most a set of λ-presentables
- it is closed under λ-directed colimits
- every object is a λ-directed colimit of λ-presentables

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The Löwenheim-Skolem Property ensures that models in an AEC are generated as directed unions of their submodels of size LS($\mathcal{K}$).

Lemma

If $\mathcal{K}$ an AEC, then $M \in \mathcal{K}$ is λ^+-presentable iff $|M| \leq \lambda$

Theorem (L)

As a category, an AEC $\mathcal{K}$ is μ-accessible for all regular $\mu >$ LS($\mathcal{K}$), and the μ-presentable objects are precisely the models of size less than μ. Moreover, $\mathcal{K}$ is closed under directed colimits.

Added structure: accessible in all regular $\mu >$ LS($\mathcal{K}$), hence *LS − accessible* in the terminology of B-R. Also, closed under arbitrary directed colimits.

Definition

A morphism $f : M \to N$ in a category $\mathbf{C}$ is said to be λ-pure (λ regular) if for any commutative square

$$\begin{array}{ccc} M & \xrightarrow{f} & N \\ u \uparrow & & \uparrow v \\ C & \xrightarrow{g} & D \end{array}$$

in which C and D are λ-presentable, there is a morphism $h : D \to M$ such that $h \circ g = u$.

In an AEC $\mathcal{K}$, $M \hookrightarrow_{\mathcal{K}} N$ is λ-pure iff M is λ-Galois-saturated relative to N. An inclusion $M \hookrightarrow_{\mathcal{K}} \mathfrak{C}$ is λ-pure iff M is λ-Galois-saturated.

Definition

A category **C** *is weakly κ-stable if for every κ^+-presentable M and morphism $f : M \to N$, f factors as*

$$M \longrightarrow M' \longrightarrow N$$

where M' is κ^+-presentable and the map $M' \to N$ is κ-pure.

If an AEC $\mathcal{K}$ is weakly κ-stable, then for any $M \in \mathcal{K}_\kappa$, the inclusion $M \hookrightarrow_{\mathcal{K}} \mathfrak{C}$ factors through a κ^+-presentable object M' (i.e. a model $M' \in \mathcal{K}_\kappa$) such that $M' \hookrightarrow_{\mathcal{K}} \mathfrak{C}$ is λ-pure, whence M' is saturated.

That is, every $M \in \mathcal{K}_\kappa$ has a saturated extension $M' \in \mathcal{K}_\kappa$.

The partial spectrum result for weakly tame AECs becomes:

Proposition

If $\mathcal{K}$ is λ-t.t., and weakly κ-stable with cf(κ) $> \lambda$, $\mathcal{K}$ is κ-stable.

As it happens, any accessible category—hence any AEC—is weakly stable in many cardinalities:

Theorem (R)

Let **C** *be a λ-accessible category, and μ a regular cardinal such that $\lambda \trianglelefteq \mu$ and $|\mathbf{Pres}_\lambda(\mathbf{C})^{mor}| < \mu$. Then* **C** *is weakly $\mu^{<\mu}$-stable.*

Taken together, these yield new partial spectrum results for weakly tame AECs. . .

Questions

Does λ-Galois-stability imply weak λ-stability?

- True in case $\mathcal{K}$ is an *elementary* class...

Can one (meaningfully) extend Galois types and the associated machinery to more general category-theoretic frameworks:

- concrete accessible categories with directed colimits,
- accessible categories with directed colimits,
- accessible categories,

and so on?

Suppose $\mathcal{K}$ is λ-categorical, C is the unique structure of size λ, and M is its monoid of endomorphisms.

Theorem (R,L)

*If $\mathcal{K}$ is λ-categorical, the sub-AEC $\mathcal{K}_{\geq\lambda}$ consisting of models of size at least λ is equivalent to (M^{op},λ^+)-**Set**, the full subcategory of M^{op}-**Set** consisting of λ^+-directed colimits of M.*

The equivalence is induced by the composition

$$\mathcal{K}_{\geq\lambda} \xrightarrow{y} \mathbf{Set}^{(\mathcal{K}_{\geq\lambda})^{op}} \xrightarrow{r} \mathbf{Set}^{M^{op}} \longrightarrow M^{op}\text{-}\mathbf{Set}$$

where y is the Yoneda embedding, and the second map is restriction.

The assignment is:

$$N \in \mathcal{K}_{\geq\lambda} \quad \mapsto \quad \mathrm{Hom}_{\mathcal{K}}(C, N)$$

where $M = \mathrm{Hom}_{\mathcal{K}}(C, C)$ acts by precomposition.

That this gives the desired equivalence is an exercise in definitions.

This amounts to an astonishing transformation of a very abstract entity—an AEC—into a category of relatively simple algebraic objects.

Questions

Does this do anything to clarify the status of the Categoricity Conjecture for AECs?

Do analogues of the Categoricity Conjecture hold for

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Do analogues of the Categoricity Conjecture hold for

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and so on? Seems unlikely, but counterexamples are very hard to come by...

Further Reading

Accessible Categories and AECs:

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Further Reading II

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