A CLASSIFICATION OF GENERALIZED ROTATIONS IN $\mathbb{C}^3$.

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Abstract. We analyse nonlinear rigid symmetries of hypersurfaces of finite Catlin multitype in $\mathbb{C}^3$. The results provide a complete classification of such manifolds. They are formulated in terms of the structure of chains for nonlinear holomorphic vector fields.

1. Introduction

One of the central problems in CR geometry is the classification of real hypersurfaces up to biholomorphic equivalence. A complete solution of this problem should also lead to a complete understanding of automorphism groups of such manifolds.

When the hypersurface is Levi nondegenerate, the problem is well understood, thanks to the classical work of Chern and Moser [11]. In particular, the infinitesimal automorphisms of such manifold form a graded Lie algebra with at most 5 components. Moreover, by results of Beloshapka, if the manifold is not equivalent to the sphere, there are at most 3 graded components, and all infinitesimal automorphisms are linear in appropriate coordinates.

In a recent paper of the authors and D. Zaitsev, the same problem is considered for Levi degenerate hypersurfaces with weighted homogeneous polynomial models in the sense of finite multitype, which replace the model hyperquadric from the nondegenerate case.

All possible structures of symmetry algebras of Levi degenerate hypersurfaces are described there. Compared to the Levi nondegenerate case, there are in general 6 possible components. The new phenomenon is the existence of nonlinear symmetries in the compex tangential variables (termed *rigid*), which are of strictly positive weight.

The local geometry of Levi degenerate hypersurfaces is in general quite complicated even on the initial level: the basic model object from the nondegenerate case - a hermitian form which defines the model

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hypersurface - is now replaced by a weighted homogeneous polynomial. More precisely, the initial invariant is captured by the notion of Catlin multitype ([9], [23], see also Section 2)

In this paper we describe completely this class of symmetries for hypersurfaces of finite multitype in $\mathbb{C}^3$.

2. Finite multitype and symmetries

Let $M \subseteq \mathbb{C}^3$ be a smooth hypersurface, and $p \in M$ be a point of finite type $m \geq 2$ in the sense of Kohn and Bloom-Graham ([1], [6], [20]).

We consider local holomorphic coordinates (z, w) vanishing at p, where $z = (z_1, z_2)$ and $z_j = x_j + iy_j$, $j = 1, 2$, and $w = u + iv$. The hyperplane $\{v = 0\}$ is assumed to be tangent to M at p, hence M is described near p as the graph of a uniquely determined real valued function

$$v = \varphi(z_1, z_2, \bar{z}_1, z_2, u), \ d\varphi(0) = 0. \tag{2.1}$$

We cay assume that (see e.g. [1])

$$\varphi(z_1, z_2, \bar{z}_1, \bar{z}_2, u) = P_m(z, \bar{z}) + o(u, |z|^m), \tag{2.2}$$

where $P_m(z, \bar{z})$ is a nonzero homogeneous polynomial of degree m without pluriharmonic terms.

Recall that the definition of multitype involves rational weights associated to the variables w, z_1, z_2. Roughly speaking, it measures the vanishing of a defining function in each of the variables

The variables w, u and v are given weight one, reflecting our choice of tangential and normal variables. The complex tangential variables (z_1, z_2) are treated according to the following definitions (for more details, see [23]).

Definition 2.1. A weight is a pair of nonnegative rational numbers $\Lambda = (\lambda_1, \lambda_2)$, where $0 \leq \lambda_j \leq \frac{1}{2}$, and $\lambda_1 \geq \lambda_2$.

Let $\Lambda = (\lambda_1, \lambda_2)$ be a weight, and $\alpha = (\alpha_1, \alpha_2)$, $\beta = (\beta_1, \beta_2)$ be multiindices. The weighted degree κ of a monomial

$$q(z, \bar{z}, u) = c_{\alpha\beta l} z^\alpha \bar{z}^\beta u^l, \ l \in \mathbb{N},$$

is defined as

$$\kappa := l + \sum_{i=1}^{2} (\alpha_i + \beta_i)\lambda_i.$$

A polynomial $Q(z, \bar{z}, u)$ is weighted homogeneous of weighted degree κ if it is a sum of monomials of weighted degree κ.

For a weight Λ, the weighted length of a multiindex $\alpha = (\alpha_1, \alpha_2)$ is defined by

$$|\alpha|_\Lambda := \lambda_1\alpha_1 + \lambda_2\alpha_2.$$

Similarly, if $\alpha = (\alpha_1, \alpha_2)$ and $\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2)$ are two multiindices, the weighted length of the pair $(\alpha, \hat{\alpha})$ is

$$|(\alpha, \hat{\alpha})|_\Lambda := \lambda_1(\alpha_1 + \hat{\alpha}_1) + \lambda_2(\alpha_2 + \hat{\alpha}_2).$$

The weighted order κ of a differential operator

$$D = \frac{\partial^{|\alpha|+|\hat{\alpha}|+l}}{\partial z^\alpha \partial \bar{z}^{\hat{\alpha}} \partial u^l}$$

is equal to

$$\kappa := l + |(\alpha, \hat{\alpha})|_\Lambda$$

Definition 2.2. A weight Λ will be called distinguished for M if there exist local holomorphic coordinates (z, w) in which the defining equation of M takes form

$$v = P(z, \bar{z}) + o_\Lambda(1), \tag{2.3}$$

where $P(z, \bar{z})$ is a nonzero Λ - homogeneous polynomial of weighted degree 1 without pluriharmonic terms, and $o_\Lambda(1)$ denotes a smooth function whose derivatives of weighted order less than or equal to one vanish.

The fact that distinguished weights do exist follows from (2.2). For these coordinates (z, w), we have

$$\Lambda = (\frac{1}{m}, \frac{1}{m}).$$

In the following we shall consider the standard lexicographic order on the set of pairs.

We recall the following definition (see [9]).

Definition 2.3. Let $\Lambda_M = (\mu_1, \mu_2)$ be the infimum of all possible distinguished weights Λ with respect to the lexicographic order. The multitype of M at p is defined to be the pair

$$(m_1, m_2),$$

where

$$m_j = \begin{cases} \frac{1}{\mu_j} & \text{if } \mu_j \neq 0 \\ \infty & \text{if } \mu_j = 0. \end{cases}$$

If none of the m_j is infinity, we say that M is of finite multitype at p.

Clearly, since the definition of multitype includes all distinguished weights, the infimum is a biholomorphic invariant.

Coordinates corresponding to the multitype weight Λ_M, in which the local description of M has form (2.3), with P being Λ_M-homogeneous, are called multitype coordinates.

Definition 2.4. Let M be given by (2.3). We define a model hypersurface M_H associated to M at p by

$$M_H = \{(z, w) \in \mathbb{C}^{n+1} \mid v = P(z, \bar{z})\}. \tag{2.4}$$

3. Nonlinear automorphisms

In this section we study nonlinear automorphisms of hypersurfaces of finite multitype a derive an explicit description of all hypersurfaces which admit a generalized rotation.

Let us first recall the following definitions.

Definition 3.1. Let X be a holomorphic vector field of the form

$$X = \sum_{j=1}^{2} f^j(z, w)\partial_{z_j} + g(z, w)\partial_w. \tag{3.1}$$

We say that X is rigid if f^1, f^2, g are all independent of the variable w.

Note that the rigid vector field W, of homogeneous weight -1, given by

$$W = \partial_w \tag{3.2}$$

lies in $\mathsf{aut}(M_P, p)$. We will denote by E the weighted homogeneous vector field of weight 0 defined by

$$E = \sum_{j=1}^{2} \mu_j z_j \partial_{z_j} + w\partial_w. \tag{3.3}$$

E is the weighted Euler field. Note that by definition of μ_j, E is a non rigid vector field lying in $\mathsf{aut}(M_P, p)$.

We can divide homogeneous rigid vector fields into three types, and introduce the following terminology.

Definition 3.2. Let $X \in \mathsf{aut}(M_P, p)$ be a rigid weighted homogeneous vector field. X is called

1. a shift if the weighted degree of X is less than zero;
2. a rotation if the weighted degree of X is equal to zero;
3. a generalized rotation if the weighted degree of X is bigger than zero and less than one.

Lemma 3.3. *Let* $X = f_1\frac{\partial}{\partial z_1} + f_2\frac{\partial}{\partial z_2}$ *be a homogeneous vector field. Then the space of homogeneous polynomials in* z *of degree* p *annihilated by* X *has complex dimension at most one.*

Proof. Suppose that P is given by

$$P(z) = \sum A_j {z_1}^j {z_2}^{p-j}, \overline{Q^{\hat{k}}}_{d_k} \tag{3.4}$$

f_1 and f_2 by

$$f_1(z) = \sum F_{1k} {z_1}^k {z_2}^{q-k}, \quad f_2(z) = \sum F_{2k} {z_1}^k {z_2}^{q-k} \tag{3.5}$$

Applying X to P, we get

$$\sum_{j,k} F_{1k} {z_1}^k {z_2}^{q-k} j A_j {z_1}^{j-1} {z_2}^{p-j} + \sum_{j,k} F_{2k} {z_1}^k {z_2}^{q-k} (p-j) A_j {z_1}^j {z_2}^{p-j-1} = 0 \tag{3.6}$$

Rewriting (3.6), we obtain

$$\sum_{j,k} (jF_{1k}A_j + (p-j+1)F_{2k}A_{j-1}) {z_1}^{(j+k)-1} {z_2}^{p+q-(j+k)} = 0, \tag{3.7}$$

with $A_{-1} := 0$.
Let k_0 be the smallest integer such that

$$\max(|F_{1k_0}|, |F_{2k_0}|) \neq 0.$$

Without loss of generality, we may assume $F_{1k_0} \neq 0$. (Otherwise we exchange the role of z_1 with z_2.)
Using (3.7), we obtain then recursively A_l as

$$lA_lF_{1k_0} = H(A_0, \ldots, A_{l-1}), \tag{3.8}$$

where H is a linear function. Since $F_{1k_0} \neq 0$, every A_l depends on A_0, and therefore the dimension of the space of such P is at most one. This achieves the proof of the lemma. □

Lemma 3.4. *Let* V_n, $n \in \mathbb{N}$, *be the space*

$$V_n = \{X | Y^n(X) = 0, \} \tag{3.9}$$

where X *is a holomorphic polynomial of a given constant weighted length and* Y *is a weighted holomorphic vector field. Then*

$$\dim V_n \leq n. \tag{3.10}$$

Moreover, when $d_n = \dim V_n > 0$, *one can choose a basis for* V_n *of the form*

(3.11)

$$\{F_s^n,\ s = 1, 2, \ldots, d_n - 1|\ Y^{d_n}(F_{d_n}^n) = 0,\ Y^{d_n-1}(F_{d_n}^n) \neq 0, Y^{d_n-1}(F_s^n) = 0\}$$

Proof. We prove the lemma by induction. The case $n = 1$ is a direct application of the previous Lemma. Suppose now that the lemma is true for n and prove it for $n + 1$. We have

$$V_{n+1} = \{X|Y^{n+1}(X) = 0\} = \{X|Y^n(Y(X)) = 0\}. \tag{3.12}$$

By induction, we obtain that

(3.13)

$$Y(X) \in \text{span}[F_s^n,\ s = 1, 2, \ldots, d_n - 1|\ Y^{d_n}(F_{d_n}^n) = 0,\ Y^{d_n-1}(F_{d_n}^n) \neq 0, Y^{d_n-1}(F_s^n) = 0]$$

which implies that

$$\dim V_{n+1} \leq n + 1. \tag{3.14}$$

After performing a linear combination of the solutions X of (3.13), we may satisfy (3.11). □

Let M_P be given by

$$M_P = \{(z, w) \in \mathbb{C}^{n+1} \mid v = P(z, \bar{z})\}. \tag{3.15}$$

Theorem 3.5. *Let* M_P *be given by* (3.15) , *and let* Y *be a generalized rotation for* M_P. *Then* P *can be decomposed in the following way*

$$P = \sum_{j=1}^{M} (\sum_{k=1}^{N_j} {Q^k}_j \overline{Q_j^{N_j-k+1}}), \tag{3.16}$$

where ${Q^k}_j$ *is a nonzero polynomial in* z *with constant weighted length* $|\alpha^k|_{\Lambda_M} =: c_k,\ c_k + c_{N_j-k+1} = 1$, *ordered such that* $c_j < c_k$ *for* $j < k$, *with*

$$\sum_{k=1}^{N_j} {Q^k}_j \overline{Q_j^{N_j-k+1}} \tag{3.17}$$

real, and

$$Y({Q^k}_j) = d_{k,j} Q_j^{k+1}, \quad d_{k,j} \in \mathbb{C}. \tag{3.18}$$

Proof. Let

$$P = \sum_{k=1}^{l} P_k, \tag{3.19}$$

where $P_1 \neq 0, P_l \neq 0$. We first consider the sum of terms in P that are of constant weighted length c_1 with respect to z. We may write them as

$$P_1 = \sum_{j=1}^{r} S_j^{c_1} \overline{S_j^{\hat{c}_1}}, \tag{3.20}$$

with r minimal. We claim that $r = 1$. Since Y is a generalized rotation, we must have

$$\overline{Y}(\sum_{j=1}^{r} S^{c_1}{}_j \overline{S_j^{\hat{c}_1}}) = \sum_{j=1}^{r} S_j^{c_1} \overline{Y}(\overline{S_j^{\hat{c}_1}}) = 0. \tag{3.21}$$

Since r is minimal, this implies that

$$Y(S_j^{\hat{c}_1}) = 0. \tag{3.22}$$

Using Lemma 3.4, we conclude, using (3.22), that

$$S_j^{\hat{c}_1} \in [S_1^{\hat{c}_1}] \tag{3.23}$$

for some $S_1^{\hat{c}_1}$ holomorphic polynomial of weight $\hat{c}_1$. We may then write P_1 as

$$P_1 = Q_1^{c_1} \overline{Q_1^{\hat{c}_1}}. \tag{3.24}$$

Hence, $r = 1$ and the claim is proved. We consider the sum of terms in P that are of constant weighted length c_k with respect to z such that $c_k = c_{k-1} + \nu$, where $\nu > 0$ is the weight of Y. We may write them as

$$P_k = \sum_{j=1}^{r} S_j^{c_k} \overline{S_j^{\hat{c}_k}}, \tag{3.25}$$

with r_k minimal. We claim that P_k can be rewritten as

$$P_k = Q_k^{c_k} \overline{Q_k^{\hat{c}_k}} + \tilde{P}_k \tag{3.26}$$

such that there is a $d_k \leq k$ such that

$$\overline{Y^{d_k}}(\overline{Q_k^{\hat{c}_k}}) = 0, \ \overline{Y^{d_k - 1}}(\overline{Q_k^{\hat{c}_k}}) \neq 0, \ \overline{Y^{d_k - 1}}(\tilde{P}_k) = 0. \tag{3.27}$$

We prove the claim by induction. The case $k = 1$ has just been proved.

Suppose by induction that (3.26) holds for k. Since Y is a generalized rotation, we have

$$Y(Q^{c_k}{}_k)\overline{Q^{\hat{c_k}}}_k + Y(\tilde{P}_k) + \sum_{j=1}^{r_{k+1}} S^{c_{k+1}}{}_{k+1} \overline{Y}(\overline{S_{k+1}^{\hat{c_{k+1}}}}) = 0. \tag{3.28}$$

Applying $\overline{Y}^{d_k}$ to (3.28), we get

$$\sum_{j=1}^{r_{k+1}} S_k^{c_{k+1}} \overline{Y^{d_k+1}} (\overline{S_{k+1}^{\hat{c_{k+1}}}}) = 0. \tag{3.29}$$

Since r_{k+1} is minimal,

$$\overline{Y^{d_k+1}} (\overline{S_{k+1}^{\hat{c_{k+1}}}}) = 0 \tag{3.30}$$

for all j. Using Lemma 3.4, we obtain that $r_{k+1} \leq d_k + 1 \leq k + 1$. Using (3.11), we may then rewrite P_{k+1} in the form given by (3.26). The claim is then proved. Let $t_1 \leq l$ be minimal such that

$$Y(Q_k^{c_k}) \neq 0,\ k = 1, \ldots, t_1 - 1, \quad Y({Q^{c_{t_1}}}_{t_1}) = 0.\}$$

We consider the following set S_1 given by

$$S_1 = \{{Q^{c_k}}_k \overline{{Q^{\hat{c_k}}}}_k, \quad k = 1, \ldots, t_1\} \tag{3.31}$$

Note that this set is not empty since $Y(Q_l^{c_l}) = 0$.

We claim that the following holds for every element of S_1.

- $d_k = k,\ k = 1, \ldots, t_1$.
- $Y({Q^{c_k}}_k) = a_k Q_{k+1}^{c_{k+1}}$,
- $Y({Q^{\hat{c_{k+1}}}}_{k+1}) = b_{\hat{k+1}} {Q^{\hat{c_k}}}_k + R_k$, where $Y^{k-1}(R_k) = 0$.

We show that $d_k = k$ using induction as above. Indeed, suppose that this is true for $k < t_1 - 1$ and show that this true for $k + 1$. Using the fact that Y is a generalized rotation, we have as in (3.28)

$$Y(Q_k^{c_k}) \overline{Q_k^{\hat{c_k}}}_k + Y(\tilde{P_k}) + ({Q^{c_{k+1}}}_{d_{k+1}}) \overline{Y} (\overline{{Q^{\hat{c_{k+1}}}}_{d_{k+1}}}) + \overline{Y}(\tilde{P_{k+1}}) = 0. \tag{3.32}$$

Applying $\overline{Y}^{k-1}$ to (3.32), we obtain

$$Y(Q_k^{c_k}) \overline{Y}^{k-1} (\overline{Q_k^{\hat{c_k}}}) + ({Q^{c_{k+1}}}_{d_{k+1}}) \overline{Y}^k (\overline{{Q^{\hat{c_{k+1}}}}_{d_{k+1}}}) = 0. \tag{3.33}$$

Hence, using (3.33), $d_{k+1} = k + 1$ by definition of S_1, and hence

$$Y({Q_k^{c_k}}_k) = a_k Q_{k+1}^{c_{k+1}}. \tag{3.34}$$

$$Y^k({Q^{\hat{c_{k+1}}}}_{k+1}) = b_{\hat{k+1}} Y^{k-1} {Q^{\hat{c_k}}}_k, \tag{3.35}$$

which implies

$$Y^{k-1}(Y({Q^{\hat{c_{k+1}}}}_{k+1}) - b_{\hat{k+1}} {Q^{\hat{c_k}}}_k) = 0, \tag{3.36}$$

and hence

$$Y({Q^{\hat{c_{k+1}}}}_{k+1}) = b_{\hat{c_{k+1}}} {Q^{\hat{c_k}}}_k + R_k, \tag{3.37}$$

where $Y^{k-1}(R_k) = 0$. This achieves the proof of the claim. Using (3.37) and (3.26), we may then assume without loss of generality that $R_k = 0$. Putting for

$$\begin{cases} Q_1^k := Q_k^{c_k}, \\ Q_1^{t_1+k} := Q_k^{\hat{c_{t_1-k+1}}}, \\ N_1 = 2t_1. \end{cases} \tag{3.38}$$

we obtain for $k = 1, \ldots, N_1$

$$Y({Q^k}_1) = d_{k,1} {Q^{k+1}}_1, \quad d_{k,1} \in \mathbb{C}, \quad d_{t_1,1} = 0. \tag{3.39}$$

In other words, we may write

$$P_k = Q_1^k \overline{Q_1^{\hat{k}}} + \tilde{P}_k, \quad k = 1, \ldots, t_1, \tag{3.40}$$

where $\tilde{P}_k$ is given by (3.26). If $t_1 < l$, we claim that the following holds:

$$d_k < k, \; k = t_1 + 1, \ldots, l. \tag{3.41}$$

Indeed, using (3.32), we obtain

$$Y(\tilde{P_{t_1}}) + ({Q^{c_{t_1+1}}}_{d_{t_1+1}}) \overline{Y}(\overline{{Q^{\hat{c_{t_1+1}}}}_{d_{t_1+1}}}) + \overline{Y}(\tilde{P_{t_1+1}}) = 0. \tag{3.42}$$

Applying $\overline{Y}^{t_1-1}$ to (3.42), we obtain that

$$\overline{Y}^{t_1}(\overline{{Q^{\hat{c_{t_1+1}}}}_{d_{t_1+1}}}) = 0, \tag{3.43}$$

and hence

$$d_{t_1+1} < t_1 + 1, \tag{3.44}$$

which implies, using the same process, that

$$d_k < k, \; k = t_1 + 1, \ldots, l.$$

This achieves the proof of the claim. We now consider

$$P_{1,k} := \begin{cases} \tilde{P}_k, \; k = 1, \ldots t_1 \\ P_k, \; k = t_1 + 1, \ldots, l \end{cases} \tag{3.45}$$

Using (3.41), we obtain

$$\overline{Y^{k-1}}(P_{1,k}) = 0. \tag{3.46}$$

We may then apply the technic we used for P_k to $P_{1,k}$ to define a set S_2 as in (3.31), and define a chain as in (3.38). Hence, after a finite number of steps, using the same process as above, we will reach the conclusion of the theorem. □

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